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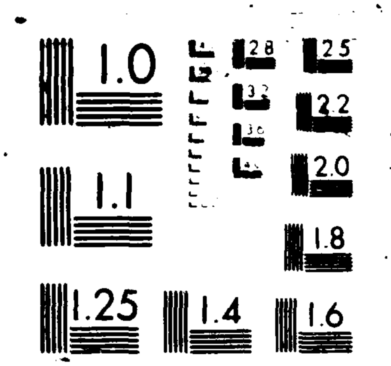

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ON THE LEAST SQUARES ESTIMATOR IN
MOVING AVERAGE MODELS OF ORDER ONE

H. A. Niroomand Chapeh & M. Bhaskara Rao
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AD-A-186028

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A simple expression is derived in this paper for the error sum of squares in the context of moving average models of order one. A computer program is developed to estimate the parameter of a moving average model of order one based on the method of least squares. (Keywords: time series analysis; consistency; algorithms; ...; multivariate analysis).

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ON THE LEAST SQUARES ESTIMATOR IN MOVING AVERAGE MODELS OF ORDER ONE

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University of Sheffield, UK

Keywords : Moving average model; Least squares estimator; Consistency

Language

Fortran 77

Description and purpose

Given a time series data, estimate β in the moving average model $Y_t = e_t + \beta e_{t-1}$, $t = 1,2,3,\ldots$ of order one using the method of least squares.

Theory

Let $Y_t = e_t + \beta e_{t-1}$, $t = 1,2,3,\ldots$ be a moving average process of order one, where $e_o = 0$ and $e_1, e_2, \ldots$ is a sequence of independent identically distributed random variables with mean zero and variance σ^2. There are several methods of estimation are available in the literature for estimating β for a given time series data $y_1, y_2, \ldots, y_N$. McClave (1974) compared the performance of some five estimators of β for moderate sample sizes by simulating the process Y_t, $t = 1,2,\ldots,100$. All these methods use approximations of one kind or the other and involve selection of some indices for a good degree of approximation. Consequently, the execution of these methods looks complex and it is natural to search for a simple method which performs competitively well with these methods. The method of least square is intuitively very appealing, and recently, under some conditions, Macpherson and Fuller (1983) showed that the least squares estimator is consistent for β in [-1,1]. We have compared the performance of least squares estimator for moderate sample sizes vis-a-vis with the five estimators examined by McClave (1974). See Niroomand Chapeh and Bhaskara Rao (1987). The least squares estimator comes out better than these five estimators. Additionally, as the present article demonstrates, the execution of the least squares method is much simpler than these five methods.

Following Macpherson and Fuller (1983), the least squares estimator of β is that value of θ in $[-1,1]$ that minimizes

$$Q_N(\theta) = \Sigma_{t=1}^{N} [e_t(Y:\theta)]^2,$$

where $e_t(Y:\theta) = Y_t - \theta e_{t-1}(Y:\theta)$, $t = 1,2,\ldots,N$ and $e_o(Y:\theta) = 0$. It works out that

$$e_k(Y:\theta) = Y_k - \theta Y_{k-1} + \theta^2 Y_{k-2} - \ldots + (-1)^{k-1}\theta^{k-1} Y_1,$$

$k = 1,2,\ldots,N$. Consequently, $Q_N(\theta)$ is a polynomial in θ of degree $2N-2$. More explicitly, for N even,

$$\begin{aligned}
Q_N(\theta) = {} & \Sigma_{i=1}^{N} Y_i^2 + (\Sigma_{i=1}^{N-1} Y_i^2 + 2\,\Sigma_{i=1}^{N-2} Y_i Y_{i+2})\theta^2 + \\
& (\Sigma_{i=1}^{N-2} Y_i^2 + 2\,\Sigma_{i=1}^{N-3} Y_i Y_{i+2} + 2\,\Sigma_{i=1}^{N-4} Y_i Y_{i+4})\theta^4 + \ldots + \\
& (\Sigma_{i=1}^{N/2+1} Y_i^2 + 2\,\Sigma_{i=1}^{N/2} Y_i Y_{i+2} + \ldots + 2\,\Sigma_{i=1}^{2} Y_i Y_{i+N-2})\theta^{N-2} + \\
& (\Sigma_{i=1}^{N/2} Y_i^2 + 2\,\Sigma_{i=1}^{N/2-1} Y_i Y_{i+2} + \ldots + 2\,\Sigma_{i=1}^{1} Y_i Y_{i+N-2})\theta^{N} + \\
& (\Sigma_{i=1}^{N/2-1} Y_i^2 + 2\,\Sigma_{i=1}^{N/2-2} Y_i Y_{i+2} + \ldots + 2\,\Sigma_{i=1}^{1} Y_i Y_{i+N-4})\theta^{N+2} + \\
& \ldots \ldots + Y_1^2\theta^{2N-2} - 2(\Sigma_{i=1}^{N-1} Y_i Y_{i+1})\theta
\end{aligned}$$

$$- 2(\Sigma_{i=1}^{N-2} Y_iY_{i+1} + \Sigma_{i=1}^{N-3} Y_iY_{i+3})\theta^3 - \dots$$

$$- 2(\Sigma_{i=1}^{N/2} Y_iY_{i+i} + \Sigma_{i=1}^{N/2-1} Y_iY_{i+3} + \dots + \Sigma_{i=1}^{1} Y_iY_{i+N-1})\theta^{N-1}$$

$$- 2(\Sigma_{i=1}^{N/2-1} Y_iY_{i+1} + \Sigma_{i=1}^{N/2-2} Y_iY_{i+3} + \dots + \Sigma_{i=1}^{1} Y_iY_{i+N-3})\theta^{N+1}$$

$$- \dots - 2Y_1Y_2\theta^{2N-3}.$$

The above expression looks formidable to include in a computer program. However, the above can be simplified as follows. Let A be the matrix of order NxN defined by

$$A = \begin{bmatrix} Y_1 & 0 & 0 & 0 & \dots & 0 & 0 \\ Y_2 & -Y_1 & 0 & 0 & \dots & 0 & 0 \\ Y_3 & -Y_2 & Y_1 & 0 & \dots & 0 & 0 \\ Y_4 & -Y_3 & Y_2 & -Y_1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{N-1} & -Y_{N-2} & Y_{N-3} & -Y_{N-4} & & (-1)^{N-2}Y_1 & 0 \\ Y_N & -Y_{N-1} & Y_{N-2} & -Y_{N-3} & \dots & (-1)^{N-2}Y_2 & (-1)^{N-1}Y_1 \end{bmatrix}.$$

Let $\underset{\sim}{\theta}$ be the column vector of order $Nx1$ defined by

$$\underset{\sim}{\theta}^T = [1, \theta, \theta^2, \theta^3, \dots, \theta^{N-1}],$$

where T stands for operation transpose. It can be verified that

$$Q_N(\theta) = \underset{\sim}{\theta}^T(A^TA)\underset{\sim}{\theta}.$$

We use this simple form of $Q_N(\theta)$ in the computer program to estimate θ.

A case study

We used the method of least squares to fit a first order moving average model to the first order differences of the time series data "IBM Common Stock Closing Prices - Daily, 17th May, 1961 to 2nd November, 1962". See Box and Jenkins (1976, p.239 and p.526). The fitted model works out to be

$$\nabla Y_t = e_t + 0.089e_{t-1},$$

$\nabla Y_t = Y_t - Y_{t-1}$ and Y_i's are the original series. Box and Jenkins obtained the estimate of β to be 0.09.

Structure

SUBROUTINE ZXMWD(FCN,M,NSIG,A1,B1,NSRCH,X,F,WORK,IWORK,IER)

Formal parameters

M	Integer	Input: number of unknowns parameters
NSIG	Integer	Input: number of digits of accuracy required in the parameter estimates
A1, B1	Real arrays	Constraint vectors of length of M Input: X(I) is required to satisfy - A1(I).LE.X(I).LE.B1(I)
NSRCH	Integer	Number of starting points to be generated Input: suggested value = min(2**M+5,100)
X	Real array	Vector of length M containing the final parameter estimates (output)
F	Real	Value of the function at the final parameter estimates (output)
WORK	Real array	Work vector of length M*(M+1)/2+11*M
IWORK	Integer array	Work vector of length M
IER	Real	Error parameter (output)

Terminal error

IER=129 indicates that the algorithm has converged to a point which may only be a saddle point

IER=130 indicates that it was not possible to calculate the solution to NSIG digits (See remarks)

IER=131 indicates that iteration was terminated after 200*(N+1) function evaluations (See remarks)

IER=132 indicates that A1(I).GE.B1(I) for some I = 1,2,...,N. No attempt is made to find the minimum in this case.

Remarks — When IER is returned as 130 or 131, the parameter estimates in X may not be reliable. Futher checking should be performed. Use of a larger NSRCH may produce more reliable parameter estimates.

Auxiliary algorithms

SUBROUTINE ZXMWD will use the FCN(M,X,F) to minimize F

M	Integer	Input: number of unknown parameters
X	Real array	Vector of length M containing the final parameter estimates (output)
F	Real	Output: Value of the function at the final parameter estimates

SUBROUTINE VMULFF(AMATT,AMAT,L,KA,KB,IA,IB,C,IC,IER) will find the multiplication of two matrices AMATT and AMAT

Formal parameters

AMATT	Real array	Input: N by N matrix stored in full storage mode
AMAT	Real array	Input: N by N matrix stored in full storage mode
L	Integer	Input: number of rows in AMATT
KA	Integer	Input: number of columns in AMATT
KB	Integer	Input: number of columns in AMAT
IA	Integer	Input: row dimension of matrix AMATT
IB	Integer	Input: row dimension of matrix AMAT

C	Real array	Output: N by N matrix containing the product C = AMATT*AMAT
IC	Integer	Input: row dimension of matrix C
IER	Integer	Output: Error indicator IER=129 indicates that AMATT or AMAT or C was dimensioned incorrectly

References

Box, G.E.P. and Jenkins, G.M. (1976) Time Series Analysis : Forecasting and Control. Holden-Day, Oakland, California

Macpherson, B.D. and Fuller, W.A. (1983) - Consistency of the least squares estimator of the first order moving average parameter, Ann.Stat., 11,326-329.

McClave, J.T. (1974) - A comparison of moving average estimation procedures, Comm.Stat., 3, 865-883.

Niroomand Chapeh, H.A. and Bhaskara Rao, M. (1987) - A comparison of various methods of estimation in moving average models (in preparation).

```
C       THIS PROGRAM WILL FIT THE FIRST ORDER MOVING AVERAGE MODEL TO DATA
        INTEGER     IER,N,L.KA,KB,IA,IB,IC,NSIG,NSRCH,IWORK( ),M,IV(N=
        REAL        X(1),A1,B1,WORK(12),F,AMAT(N=,N= ,AMATT(N=,N= ,C(N=,N=
        EXTERNAL    FCN
        COMMON      N,C
        F1=0.0
        F2=0.0
        OPEN(UNIT=100, FILE='DATA.DAT',STATUS='OLD')
        OPEN(UNIT=99, FILE='RES.DAT', STATUS='NEW')
        READ(100,*)  IV
        M=1
        NSIG=4
        NSRCH=7
        A1=1.0
        B1=1.0
        N=
        L=N
        KA=N
        KB=N
        IA=N
        IB=N
        IC=N
C       THE GIVEN DATA IS PUT INTO THE MATRIX A AS EXPLAINED IN THE THEORY
        DO     2     J=1,N
        DO     2     I=J,N
```

```
      AMATT I,J = -1** J-1 *IV(I-J+1)
1     CONTINUE
      DO      I       I=1,N
      D               J=1,N
      AMATT I,J =AMAT I,J
      CONTINUE
      CALL    VMULFF AMATT,AMAT,L,KA,KB,IA,IB,C,IC,IER)
      CALL    ZXMWD FCN,M,NSIG,A1,B1,NSRCH,X,F,WORK,IWORK,IER)
      WRITE 6,101       X 1
101   FORMAT 'FINAL RES IS := AHAT', F10.4)
      STOP
      END
      SUBROUTINE FCN M,X,F
      INTEGER     M,I,J,N
      REAL        X 1 ,F,C N=,N=
      COMMON      N,C
      F1=0.0
      DO     200     J=0,N-1
      SUM1=0.0
      DO     20     I=1,J+1
20    SUM1=SUM1+C I,J+1-I
200   F1=F1+ X 1 **J *SUM1
      F2=0.0
      DO     300     J=2*N-2,N,-1
```

```
      SUM2=0.0
      DO 30 I=J+2-N,N
30    SUM2=SUM2+C(I,J+2-I)
300   F2=F2+(X(1)**J)*SUM2
      F=F1+F2
      RETURN
      END
```

Concluding remarks : This work is useful for drawing inferences for signal process models when the observations form a moving average model of order one. A manuscript is under preparation detailing applications of the results of this paper to signal processing.

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